EFFICIENCY OF MULTILAYER RECTANGULAR FINS

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We solve the heat-conduction problem for a multilayer rectangular transverse fin on a circular tube. We investigate the effect of fin dimensions on fin efficiency.

The fin efficiency E characterizes the temperature variation over the surface of a fin. In most cases the air-coolers of refrigerating units are covered with a layer of frost. The distribution of temperatures on such a "three-layer fin" can be obtained by solving a system of heat-conduction equations. The following simplifications are usually made: 1) the material of the fin is homogeneous and isotropic; 2) the heat flux to or from the fin at each point of its surface is directly proportional to the difference between the fin temperature at that point and the temperature of the medium flowing past it; 3) the thermal conductivity of the fin is constant; 4) there are no heat sources in the fin itself; 5) the coefficient of heat transfer to the fin is constant over the entire surface of the fin; 6) the temperature at the base of the fin is constant; 7) the temperature of the surrounding medium is constant; 8) the amount of heat passing through the outer edge of the fin is negligible in comparison with the amount of heat transferred through the lateral surfaces; 9) the thickness of the frost layer is constant over the entire fin surface; 10) there is no contact thermal resistance between the layers.

Some approximate equations for calculating the efficiency of rectangular and square fins covered with a frost layer are already known [1]:

$$E=\frac{th\,(mh)}{mh};\; m=\sqrt{\frac{2}{\left(\frac{1}{\alpha\xi}+\frac{\delta_r}{\lambda_r}\right)\lambda_{fi}\delta_{fi}}};\tag{1}$$

$$h=0.5d_e(\rho-1)(1+0.805\lg\rho);\quad \rho=1.28\frac{s_{t\,min}}{d_e}\sqrt{\frac{s_{t\,max}}{s_{t\,min}}-0.3}.$$

For $mh \geqslant 2.7$ (see [2])

$$E=1/(mh).\tag{2}$$

In the optimization of refrigerating units, when we select the optimal fin arrangement, an error in determining E results in an almost equal error in determining the optimality factor. The values of the efficiency E found for square fins in accordance with (1) differ from the experimental values [4] by 5-40%. The method proposed below makes it possible to obtain more accurate values.

By virtue of simplifications 1-10 and considerations of symmetry, the problem reduces to a "two-layer" stationary heat-conduction problem in the region illustrated in Fig. 1.

Let $V = T_{fr} - T_a$ be a function given on the frost layer; let $U = T_{fi} - T_a$ be a function given on the metal layer. We are required to solve the system of differential equations

$$\begin{cases}\dfrac{\partial^2 U}{\partial x^2}+\dfrac{\partial^2 U}{\partial y^2}+\dfrac{\partial^2 U}{\partial z^2}=0,\\[2mm] \dfrac{\partial^2 V}{\partial x^2}+\dfrac{\partial^2 V}{\partial y^2}+\dfrac{\partial^2 V}{\partial z^2}=0\end{cases}\tag{3}$$

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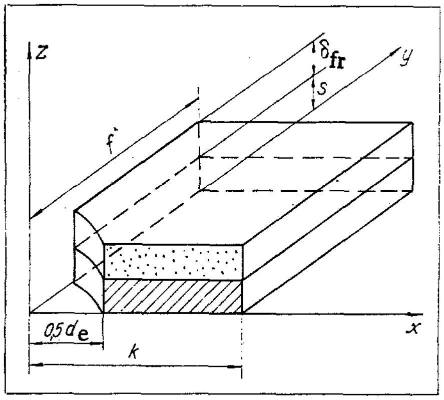

Fig. 1. Domain of definition of the functions U(x, y, z), $0 \leqslant z \leqslant s$ and V(x, y, z), $s \leqslant z \leqslant s + \delta_{fr}$.

with the boundary conditions

$$-\frac{\lambda_{fr}}{\alpha}\,\frac{\partial V}{\partial z}\Big|_{z=s+\delta_{fr}} = V\Big|_{z=s+\delta_{fr}};\quad \frac{\partial U}{\partial z}\Big|_{z=s} = \frac{\lambda_{fr}}{\lambda_{fi}}\cdot\frac{\partial V}{\partial z}\Big|_{z=s};$$

$$\frac{\partial V}{\partial x}\Big|_{x=0} = \frac{\partial U}{\partial x}\Big|_{x=0} = \frac{\partial V}{\partial y}\Big|_{y=0} = \frac{\partial U}{\partial y}\Big|_{y=0} = \frac{\partial V}{\partial x}\Big|_{x=k} = \frac{\partial U}{\partial x}\Big|_{x=k} = \frac{\partial V}{\partial y}\Big|_{y=f} = \frac{\partial U}{\partial y}\Big|_{y=f} = \frac{\partial U}{\partial z}\Big|_{z=0} = 0; \tag{4}$$

$$U|_{x^2+y^2=(0.5d_e)^2} = T_{fb} - T_a;\; U|_{z=s} = V|_{z=s}.$$

Because the boundary of the domain of definition of the functions U and V is quite complicated, we resort to the following method. We shall enlarge the domain so as to make it a complete parallelepiped and solve the following boundary-value problem: it is required to solve the system (3) with the boundary conditions

$$\frac{\partial U}{\partial y}\Big|_{y=0} = \varphi(x,\,z) = \begin{cases} w_1xz + w_2x^2 + w_3z^2 + w_4x + w_5z + w_6, \\ \qquad\qquad 0 \leqslant x \leqslant 0.5d_e, \\ 0, \qquad\quad 0.5d_e < x \leqslant k, \end{cases}$$

$$\frac{\partial V}{\partial y}\Big|_{y=0} = \psi(x,\,z) = \begin{cases} w_7xz + w_8x^2 + w_9z^2 + w_{10}x + w_{11}z + w_{12}, \\ \qquad\qquad \text{if}\;\; 0 \leqslant x \leqslant 0.5d_e, \\ 0, \qquad\quad \text{if}\;\; 0.5d_e < x \leqslant k \end{cases} \tag{4a}$$

and other boundary conditions coinciding with (4).

The functions $\varphi(x, z)$ and $\psi(x, z)$ are chosen to be polynomials of second degree on the basis of physical considerations; the parameters w_i (i = 1, 2, ..., 12) are determined after solving the boundary-value problem, requiring that the functions U and V have the values of $T_{fb} - T_{fi}$ on the surface $x^2 + y^2 = (0.5d_e)^2$, $0 \leqslant z \leqslant s + \delta_{fr}$.

Since the functions U and V are even functions of x and y and the boundary conditions are of the first and second kinds, in order to solve the boundary-value problem we can use the Fourier cosine transform successively with respect to x and y [3]. Taking account of the boundary conditions (4), (4a), we arrive at a system of ordinary differential equations of second order with constant coefficients:

$$\frac{d^2\overset{\sim y}{\overset{\sim x}{U}}(p,\,q,\,z)}{dz^2} - a^2\overset{\sim y}{\overset{\sim x}{U}}(p,\,q,\,z) = \tilde{\varphi}^x(p,\,z),$$

$$\frac{d^2\overset{\sim y}{\overset{\sim x}{V}}(p,\,q,\,z)}{dz^2} - a^2\overset{\sim y}{\overset{\sim x}{V}}(p,\,q,\,z) = \tilde{\psi}^x(p,\,z) \tag{5}$$

with boundary conditions

$$\frac{d\overset{\sim y}{\overset{\sim x}{V}}}{dz}\Big|_{z=s+\delta_{fr}} + \frac{\lambda_{fr}}{\alpha}\overset{\sim y}{\overset{\sim x}{V}}\Big|_{z=s+\delta_{fr}} = 0;\quad \frac{d\overset{\sim y}{\overset{\sim x}{U}}}{dz}\Big|_{z=0} = 0;$$

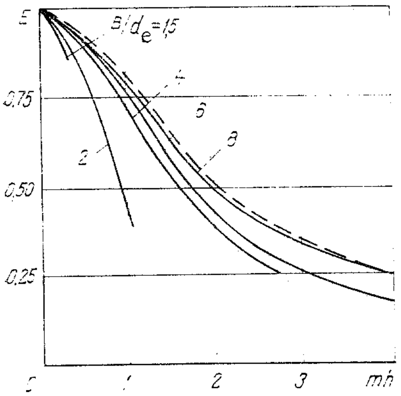

Fig. 2

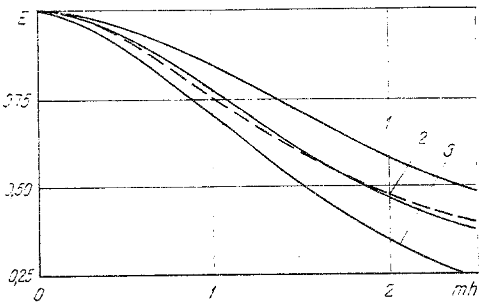

Fig. 3

Fig. 2. Efficiency of square fins as a function of mh for various values of B/d_e. The dashed curve corresponds to the values obtained from (1).

Fig. 3. Effect of fin shape on efficiency [1) $r_{fi} = 3l_{fi}/2$; 2) $r_{fi} = l_{fi}$; 3) $r_{fi} = 2l_{fi}/3$]. The dashed curve corresponds to the values obtained by (1). $l_{fi}/d_e = 10$.

$$\left.\frac{d\overset{\sim y}{\overset{\sim x}{U}}}{dz}\right|_{z=s} = \frac{\lambda_{fr}}{\lambda_{fi}} \cdot \left.\frac{d\overset{\sim y}{\overset{\sim x}{V}}}{dz}\right|_{z=s}; \quad \overset{\sim y}{\overset{\sim x}{U}}\big|_{z=s} = \overset{\sim y}{\overset{\sim x}{V}}\big|_{z=s}. \tag{6}$$

Here $\overset{\sim y}{\overset{\sim x}{U}}$, $\overset{\sim y}{\overset{\sim x}{V}}$ are cosine transforms of the functions U and V, respectively, with respect to x and y. The quantities $\overset{\sim x}{\varphi}$, $\overset{\sim x}{\psi}$ are the cosine transforms of φ and ψ with respect to x.

For fixed p and q and $a \neq 0$ the system (5) has the general solution

$$\overset{\sim y}{\overset{\sim x}{U}}(p,\ q,\ z) = C_1(p,\ q)\exp(az) + C_2(p,\ q)\exp(-az) + R(p,\ q,\ z),$$

$$\overset{\sim y}{\overset{\sim x}{V}}(p,\ q,\ z) = C_3(p,\ q)\exp(az) + C_4(p,\ q)\exp(-az) + Q(p,\ q,\ z).$$

For $a = 0$

$$\overset{\sim y}{\overset{\sim x}{U}}(0,\ 0,\ z) = C_1(0,\ 0)z + C_2(0,\ 0) + R(0,\ 0,\ z)z^2,$$

$$\overset{\sim y}{\overset{\sim x}{V}}(0,\ 0,\ z) = C_3(0,\ 0)z + C_4(0,\ 0) + Q(0,\ 0,\ z)z^2,$$

where the $C_j(p, q)$ (j = 1, 2, 3, 4) are determined from the boundary conditions and R(p, q, z), Q(p, q, z) are polynomials of second degree in z.

The functions U and V can be written in the form of a series ([3])

$$U(x,\ y,\ z) = \frac{1}{kf}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\overset{\sim y}{\overset{\sim x}{U}}(p,\ q,\ z)\cos\frac{\pi px}{k}\cos\frac{\pi qy}{f},$$
$$V(x,\ y,\ z) = \frac{1}{kf}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\overset{\sim y}{\overset{\sim x}{V}}(p,\ q,\ z)\cos\frac{\pi px}{k}\cos\frac{\pi qy}{f}. \tag{7}$$

The series (7) are majorized by the convergent series $\sum_{n=1}^{\infty} U_n$ with the general term $U_n = 1/(p^2 + q^2)$ and are therefore convergent.

It is usually sufficient to use a few terms of the series ($p \leqslant 3$, $q \leqslant 3$) in order to obtain a result with three accurate digits after the decimal point. The average of the function U(x, y, z) in the domain D (at the interface between the metal and the frost) can be found from the formula

TABLE 1. Efficiency Values of Square Transverse Fins Covered with a Layer of Frost (d_e = 0.012 m, T_{fb} = 267°K, T_a = 272°K)

l_{fi}	r_{fi}	δ_{fi}	λ_{fi}	δ_{fr}	λ_{fr}	mh	E
0,024	0,024	0,0002	109,3	0,0002	0,116	0,2	0,942
0,024	0,024	0,0002	109,3	0,0008	0,116	0,2	0,943
0,048	0,048	0,0002	109,3	0,0002	0,116	0,2	0,982
						0,5	0,895
						2,0	0,334
0,048	0,048	0,0002	109,3	0,0008	0,116	0,2	0,982
						0,5	0,902
						2,0	0,358
0,096	0,096	0,0002	109,3	0,0002	0,116	0,2	0,988
						1,0	0,777
						1,5	0,626
						2,5	0,397
0,096	0,096	0,0002	109,3	0,0002	0,116	0,2	0,988
						1,0	0,782
						1,5	0,611
						2,5	0,377
0,1	0,1	0,0002	116,3	0,001	1,163	0,2	0,99
						0,5	0,933
						1,0	0,795
						1,5	0,620
						2,5	0,372
0,1	0,1	0,0002	116,3	0,001	0,233	0,2	0,987
						0,5	0,935
						1,0	0,781
						1,5	0,610
						2,5	0,368
0,1	0,1	0,0002	116,3	0,001	0,116	0,2	0,988
						0,5	0,932
						1,0	0,770
						1,5	0,622
						2,5	0,350
						2	0,476
0,1	0,1	0,0002	116,3	0,001	0,002	0,2	0,982
						0,5	0,930
0,1	0,1	0,002	116,3	0,001	0,116	0,2	0,988
						0,5	0,932
						1,0	0,778
						1,5	0,610
						2,0	0,480
0,1	0,1	0,016	116,3	0,001	0,116	0,2	0,988
						0,5	0,930
0,1	0,1	0,0002	23,3	0,001	0,116	0,2	0,989
						0,5	0,935
						1,0	0,780
						1,5	0,614
						2,0	0,498
0,1	0,1	0,0002	11,6	0,001	0,116	0,2	0,989
						0,5	0,938
						1,0	0,790
						1,5	0,603
						2,0	0,478
						2,5	0,410
0,1	0,1	0,0002	350	0,001	0,116	0,2	0,988
						0,5	0,932
						1,0	0,778
						1,5	0,630
						2,5	0,376

$$U_{av} = \iint\limits_{(D)} U(x, y, s)\,dxdy \Big/ \Big(\iint\limits_{(D)} dxdy\Big).$$

The fin efficiency can then be written as

$$E = U_{av}/(T_{fb} - T_a).$$

The error in the values of E found by this method is negligible in comparison with the experimental data for "single-layer" square fins [4].

As can be seen from the foregoing, the method is very cumbersome. It is almost impossible to use it for hand calculations. However, finding E by this method on high-speed electronic computers does not substantially increase the total time of the optimization calculations. For δ_{fr} = 0 we obtain a heat-conduction problem for a "single-layer" thin fin.

The methods of operational calculus can be used in solving the heat-conduction equations for other types of fin arrangement.

For our analysis, we carried out calculations for "single-layer" and "three-layer" square and rectangular fins for a wide range of variation of the parameters $l_{fi} = 2k$, $r_{fi} = 2f$, δ_{fi}, λ_{fi}, δ_{fr}, λ_{fr}, α.

The results of the calculations for some square fins covered with a frost layer are shown in Table 1. In Fig. 2 we show the results of the calculations in the form of the function $E = F(mh, B/d_e)$. Here B is the length of a fin along the path of motion of the air; for a square fin this is a side of the square. This traditional representation enables us to compare directly the results of calculations made by the proposed method and the results of calculations made by other authors, in particular the results of calculations based on formula (1).

In addition, it was found that the effects of the thickness of the fin and the frost layer and those of the thermal conductivity of the fin material and the frost are completely taken into account by the quantity mh, i.e., fins with equal values of B and mh have equal efficiency E for various values of δ_{fi}, λ_{fi}, δ_{fr}, and λ_{fr}. When we pass to other fins with other values of B, even for identical values of mh, δ_{fi}, λ_{fi}, δ_{fr}, and λ_{fr}, the efficiency changes; this is clearly visible from Table 1.

Consequently, the number of variables on which E depends is equal in practice to two for square fins:

$$E = F\left(mh,\ \frac{B}{d_e}\right)$$

and three for rectangular fins:

$$E = F_1\left(mh,\ \frac{B}{d_e},\ \frac{l_{fi}}{r_{fi}}\right).$$

In Fig. 2 we indicate by a dashed curve the values of E obtained by formula (1). This curve practically coincides with the curve for $B/d_e = 8$.

For values of $B/d_e \leqslant 6$ formula (1) yields a value which is too high: the error amounts to as much as 30%; for $B/d_e > 6$ the values are in good agreement with the calculated values: the error reaches 13% only in isolated cases.

Figure 3 shows the graphs for the efficiency E of rectangular fins as a function of mh for fins of various shapes. It can be seen from the figure that E depends very much on the ratio l_{fi}/r_{fi}, and therefore the error in calculating the efficiency by (1) can amount to as much as 54%.

Thus, the proposed method is much more accurate than previously known methods. Graphs and diagrams constructed by this method can be used for hand calculations of E.

NOTATION

α, heat-transfer coefficient from air, $W/(m^2 \cdot deg)$; ξ, moisture condensation coefficient; δ_{fi}, fin thickness, m; λ_{fi}, thermal conductivity of fin material, $W/(m \cdot deg)$; δ_{fr}, thickness of frost layer, m; λ_{fr}, thermal conductivity of frost, $W/(m \cdot deg)$; $a = \pi\sqrt{(p^2/k^2) + (q^2/f^2)}$; d_e, external diameter of finned tube, m; $f = r_{fi}/2$, m; $s = \delta_{fi}/2$, m; $k = l_{fi}/2$, m; l_{fi}, length of fin along path of motion of the air, m; r_{fi}, width of fin, m; s_{tmin}, s_{tmax}, minimum and maximum distance between tubes, m; T_{fi}, T_{fr}, T_a, T_{fb}, temperatures of fin, frost, air, and fin base, respectively, °K.

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